International Journal of Mathematics and
Computer Applications Research (IJMCAR)
ISSN(P): 2249-6955; ISSN(E): 2249-8060
Vol. 5, Issue 3, Jun 2015, 11- 24

THE NUCLEI U=C IN ALTERNATIVE PRIME RIGHT RINGS

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ABSTRACT

In this paper we have to prove that the collection of sub rings called nuclei some of which like commutative centre $U=\{u\in R/(u,R)=0\}$, $C=\{c\in U/(R,R,c)=(c,R,R)=0\}$ either equal i.e

$U=C$ or strongly (-1,1) ring when R is prime right alternative ring of characterstic≠2,3 with $(R,R,U)\subseteq U$ or $S(p^2,p,q)=0.$

KEYWORDS: Nucleus, Centre, Commutative Centre, Right Alternative Rings, Strongly (-1, 1) Ring

INTRODUCTION

Let R be a non associative ring with characteristic $\neq$ 2, 3, which have non zero locally nilpotent ideals. At first Rommel'di [1] show that U=C, particularly when R is simple right alternative ring with characteristic $\neq$2.U=C is also shown when the alternator ideal of R is either right or left nilpotent particularly when R satisfies the minimum condition on right ideals. Later in an example due to Pchelincev [2] shows that U$\neq$C when R is a prime strongly (-1, 1) ring. Pchelincev also established the nil potency of the associators in a free (-1, 1) rings. Miheev [3] had constructed a finite dimensional prime right alternative nil algebra in which $N_\lambda \neq U$ and $N_\lambda \neq N_\gamma$. Now in this paper we have to prove that when (R,R,R)$\subseteq$U then either U=C or R is strongly (-1,1) ring

K. Suvarna[4]proved some results in this direction when R is a prime right alternative ring with characteristic $\neq$2.In particular when R be a prime right alternative ring with characteristic$\neq$ 2,3 then U=C or R is strongly (-1,1) ring if either [R,R] $\subseteq N_\alpha$ or R satisfies S $(p^2,p,q)=0$.

PRELIMINARIES: Let R be a non associative ring. we shall denote the associator and commutator by

$$(pq)=pq-qp$$

$$(pqr)=(pq)r-p(qr) \text{ for all } p,q,r \text{ in R.}$$

A ring is called right alternative if it satisfies the identity (q,p,p) = 0 which also satisfies the identity (p,p,q) = 0 is called alternative and one which satisfies the identity [(p,q),r]= 0 is called strongly (-1,1).

The following are the notations are used for nuclei and centers in right alternative ring R. Left nucleus, $N_{\alpha} = \{\alpha \in R/(\alpha,\beta,\beta) = 0\}$ Right nucleus, $N_{\gamma_{\gamma}} = \{\alpha \in R/(\beta,\beta,\alpha) = 0\}$

Associative centre or Nucleus $N = N_{\alpha} \cap N_{\gamma}$

The right alternative nucleus $N_{\lambda} = \{v \in R/(P,P,V) = 0\}$

Alternative centre $N_{\rho} = \{v \in R/(p,v,p) = 0, (v,p,q) = (p,q,v) = (q,v,p)\}$

Commutative centre $U = \{u \in R/(u,R) = 0\}$

The associative commutative Centre C = $N \cap U = \{C \in N/(c,R,R) = (R,R,C) = 0.$

A right alternative ring R is said to be prime if AB=0 for ideal Aand Bof R implies either A=0 or B=0. Now we show that U=C when R is prime right alternative ring.

The identity know as Teichmuller identity holds in any non associative ring is

1). $(\omega p,q,r) - (\omega, pq, r) + (\omega, p, qr) = \omega(p,q,r) + (\omega,p,q)r.$

From this identity it is clear that, left and right nuclei are the associative sub rings of R.The following are the identities satisfied in any right alternative ring

2) $(q,p,p) = 0$

Its linearization gives $(q,p,r) + (q,r,p) = 0.$ ---------------$(2^1).$

3) $[pq,r] = p[q,r] + [p,r]q + 2(p,q,r) + (r,p,q)$

4) $2s(p,q,r) = [[p,q],r] + [[q,r],p] + [[r,p]q]$

5) $(r,p^2,q) = (r,p,pq+qp)$

6) $(r,p,pq) = (r,p,q)p$

7) $(\omega p,q,r) + (\omega,p,[q,r]) = \omega(p,q,r) + (\omega,q,r)p$

8) $([\omega,p],q,r) - [\omega,(p,q,r)] + [p,(\omega,q,r)] = (p,\omega[q,r]) - (\omega,p,[q,r])$

9) $(N_{\delta},R,R) \subseteq N_{\delta}$

10) $((a,q,r),b,c) = ((a,b,c),q,r) - (a,b,(c,q,r)) - (a,(b,q,r),c) + (a,b,c)[q,r] - (a,b,c[q,r]) + (a,b,[q,r])c$

Now we have to prove certain identities and relations involving the commutative centre U of right alternative ring R with characteristic $\neq 2$,Now $u \in U.$

11) $(u,q,p)=2(p,q,u)$

12) $((p,q,u),r,\omega)=2(\omega,r,(p,q,u))$

13) $(p,p,u)=0$

The fundamental and most extremely useful tool in non associative algebra is linearization using his concept we replace a repeated variable or an identity by the sum of two variables in order to obtain another identity like

13)[1] $(p,q,u)+(q,p,u)=0$

14)$[p(p,q,u)]=0$ its linearization leads the identity --(14)[1]

14)[1] $(p,(r,q,u)+(r,(p,q,u))=0$

15) $2[p,(r,q,u)]=[p,(u,q,r)]=(p[q,r],u)$

16) $\left(p,[p,q],u\right)=0$ its linearization leads the identity--(16)[1]

16)[1] $(p,[r,q],u)+(r,[p,q],u)=0$

17) $([p.q],[r,w],u)=0$

18) $(R,[R,R],U)\subseteq U \Leftrightarrow (R,R,U)\subseteq U$

19) $[R(R,R,U)]\subseteq U$

20) $S((p,q,u),r,\omega)=0$

21) $(a,a,(p,q,u))=0$ its linearization leads the identity--(21)[1]

21)[1] $(a,r(p,q,r)+(r,a(p,q,r))=0$

22) $3(c,a(a,q,u))=((a,a,c),q,u)$

23) $3(p,(p,r,(a,b,u))=[(p,p,r),(a,b,u)]$

The proofs related to commutative centre are from 11-23 as follows

11) $(u,q,p)=2(p,q,u)$

$\{pq,u]=p[q,u]+[p,u]q+2(p,q,u)+(u,p,q)$ ---from (3)

$2(p,q,u)+(u,p,q)=[pq,u]-p[q,u]-[p,u]q=0$

$2(p,q,u)=-(u,p,q)$

$2(p,q,u)=-(-(u,q,p))$ --from (2)[1]

$$2(p,q,u)=(u,q,p)$$

Similarly the proof of (12)

13) $(p,p,u)=0$

From the definition of right alternative ring ie from (2)

$$(u,p,p)=0$$

$(u,p,p)=2(p,p,u)$ --from (11)

Since characteristic $\neq 2$

$$(p,p,u)=0$$

14) $[p,(p,q,u)]=0$

$p(p,q,u)+(p,q,u)p=(p^2,q,u)+(p,p,[q,u])$ -------------------from(7)

$$=(p^2,q,u)$$

$$=-(q,p^2,u)$$

$=-(q,p,pu+up)$ --------------------------from (5)

$$=-[(q,p,pu)+(q,p,up)]$$

$$=-2(q,p,pu)$$

$=-2(q,p,u)p$ ------------------------------from (6)

$=2(p,q,u)p$ ---------------------------------from (13)

$$p(p,q,u)=2(p,q,u)p-(p,q,u)p$$

Thus $p(p,q,u)=(p,q,u)p$ or $[p(p,q,u)]=0$

15) $2[p,(r,q,u)]=[p,(u,q,r)]=(p,[q,r],u)$

$2[p,(r,q,u)]=[p(u,q,r)]$ --from (11)

$\Rightarrow$ Using the identity (8) by replacing $\omega=u$

$[p,(u,q,r)]=-[(u,p),q,r]+[u,(p,q,r)]+[p,u,[q,r]]-[u,p,[q,r]]$ -----from (8)

$=-[p,[q,r],u]+[u,[q,r],p]$ ---from (2^1)

$= -[p,[q,r],u] + 2[p,[q,r],u]$ --from (11)

$= (p,[q,r],u).$

16) $(p,[p,q],u) = 0$

Using the identity (15) $(p,[p,q],u) = 2[p,(q,p,u)]$

$= -2[p,(p,q,u)]$ ------------------------------from (13)[1]

$= 2(p,(p,q,u)] = 0$ ---------------------------from (14)[1]

Since the characterstic≠2 $\Rightarrow (p,[p,q],u) = 0$

17) $([p,q],[r,\omega],u) = 0$

Put $\omega = p, p = q, q = [r,\omega], r = u$ in (8) getting

$([p,q],[r,\omega],u) = [p,(q,[r,\omega],u)] - [q,(p,[r,\omega],u)] + (q,p,([r,\omega],u) - (p,q,[[r,\omega],u])$

$= [p,(q,[r,\omega],u)] - [q,(p,[r,\omega],u]$ From (16)[1] and (14)[1] again (16)[1]

$= -[p,(r,[q,\omega],u)] + [q,(r,[p,\omega],u)]$

$= -[p,(r,[q,\omega],u)] - [r,(q,[p,\omega],u)]$

$= -[p,(r,[q,\omega],u)] + [r,(p,[q,\omega],u)]$ --From (8)

$= -([p,r],[q,\omega],u) + (r,p,[[q,\omega],u]) - (p,r,[[q,\omega],u])$

$= -([p,r],[q,\omega],u)$

$= -([r,p],[\omega,q],u)$

$= ([r,\omega],[p,q],u)$

$= -([p,q],[r,w],u)$

$([p,q],[r,\omega],u) + ([p,q],[r,w],u) = 0$

$\Rightarrow 2([p,q].[r,\omega],u) = 0$ since characteristic $\neq 2$

$\Rightarrow ([p,q],[r,\omega],u) = 0$

18) $(R,[R,R],u) \subseteq U$

$2[p,(r,[a,b],u)] = (p,[[a,b],r],u)$ -- from (15)

$= -([a,b],[p,r],u) = 0$ ---from(16)[1]&(17)

$2[p,(r,[a,b],u] = 0$ since the characteristic ≠2 hence $(R,[R,R],u) \subseteq U$

19) $[R,(R,R,U)] \subseteq U$

$2[p,(r,q,u)] = (p,[q,r],u) \in U$ -- From (15)

$= (R,[R,R],U) \subseteq U$ --- Using (18)

$2(p,(r,q,u)] = 0$ since the characteristic ≠2

$[p,(r,q,u)] = 0$

$\Rightarrow [R,(R,R,U)] \subseteq U$

20) $S((p,q,u),r,\omega) = 0$

$2s((p,q,u),r,\omega) = [[(p,q,u),r],\omega] + [(r,\omega)(p,q,u)] + [[\omega,(p,q,u),r]$ ----------------- using (4)

$= [(r,\omega)(p,q,u)]$ ---Using (14)[1] &(19)

$= -[p,((r,\omega),q,u)]$ ---From(13)[1]

$= [p,(q,(r,\omega),u)] = 0$ since the characterstic ≠ 2

$\Rightarrow s((p,q,u),r,\omega) = 0$

21) $((a,a(p,q,u)) = 0$

$(u,q,p) = 2(p,q,u)$ --Using (11)

Since $U \subseteq N_\lambda$, $(N_\lambda,R,R) \subseteq N_\lambda$ ---From (9)

$\Rightarrow 2(p,q.u) = (u,q,p) \in N_\lambda$

Thus $2((a,a,(p,q,u)) = 0$ since characteristic ≠ 2

We have $(a,a,(p,q,u)) = 0$

22) $3(c,a(a,q,u)) = ((a,a,c),q,u)$

From (10)

$((a,a,c),q,u) = ((a,q,u),a,c) - (a,q,(u,a,c)) - (a,(q,a,c),u) + (a,q,u)[a,c] - (a,q,u[a,c]) + (a,q,[a,c])$,

$(a,a,(p,q,u)) = 0$ -- from (21)

$((a,q,u),a,c)+(c,a,(a,q,u))=0$

$=2(c,a,(a,q,u))+(c,a,(a,q,u))$ --from (12)

$=3(c,a,(a,q,u))$

23) $3[p,(p,r,(a,b,u)]=[(p,p,r)(a,b,u)]$

$([p,(u,a,b)],p,r)=2([p,(b,a,u)],p,r)$ --from(11)

$=((p,[a,b],u),p,r)$ --from (15)

$=2(r,p,(p,[a,b],u))$ ---from (12)

$([p,(u,a,b)],p,r)=2(r,p,(p,[a,b],u))$ ---------------------------------------from (24)

$([p,r],p,(a,b,u))=[p,(r,p,(a,b,u)]-[r,(p,p,(a,b,u)]+(r,p,[p,(a,b,u)]-(p,r,[p,(a,b,u)])$

Using (8) and then using (21) it reduces to

$=[p,(r,p,(a,b,u)]-0+(r,p,[p,(a,b,u)])+(r,p,[p,(a,b,u)])$

$=[p,(r,p,(a,b,u)]+2(r,p,[p,(a,b,u)])$

Since by (19) $[p,(a,b,u)]\in u\subseteq N_\lambda$ then from (15) and (13)[1] we have

$-2(r,p,[p,(a,b,u)])=(p,(r,p,(a,b,u))-([p,r],p,(a,b,u))$ ----------------------------- using (15)

$-(r,p,(p.[b,a],u)=(p,(r,p,(a,b,u))-([p,r],p,(a,b,u))$ ------------------------------- using (13)[1]

$(r,p,(p,[a,b],u))=(p,(r,p,(a,b,u))+([p,r],p,(b,a,u))$ --------------------------------- using(25)

$2(r,p,(p,[a,b],u))=([p,(u,a,b)],p,r)$ -- -from(24)

$=[p,((u,a,b),p,r)]-[(u,a,b),(p,p,r)]+((u,a,b),p,[p,r])-(p,(u,a,b)[p,r])$ ------from(8)&(26)

$=-[(u,a,b)(p,p,r)]-[(u,a,b),(p,p,r)]+((u,a,b),p,[p,r])-2([p,r],(u.a.b),p)$ using (11)&(12)

$=-2[(u,a,b)(p,p,r)]+((u,a,b),p,[p,r])+2([p,r],p,(u.a.b))$

$((u,a,b),p,[p,r])=-2([p,r],p,(u,a,b))$

$((u,a,b),p,[p,r])=-4([p,r],p,(b,a,u))$

$((u,a,b),p,[p,r])=4([p,r],p,(b,a,u))$ --------------------------------------using-(27)

$-(p,(u,a,b),[p,r]) = 2((b,a,u),p,[p,r])$ ----------------------------------using (11)

$= -2((b,a,u),[p,r],p)$ -----------------------------------using (2)[1]

$= 2([p,r],(b,a,u),p)$ ----------------------------------using (21)

$= -2([p,r],p,(b,a,u))$ --- (28)

Make use (27) &(28) in (26)

$$2(r,p,[p,[a,b],u]) = (p,(u,a,b),p,r) - ((u,a,b),(p,p,r)) + 4([p,r],p,(b,a,u)) - 2([p,r],p,(b,a,u))$$

$= (p,(u,a,b),p,r) - ((u,a,b),(p,p,r)) + 2([p,r],p,(b,a,u))$ ------------(29) Multiplying equation (25) with(2)

$2(r,p,(p,[a,b],u)) = 2[p,(r,p,(a,b,u)] + 2[[p,r],p,(b,a,u)]$ ----------------------------------(30)

Substitute R.H.S of (30) in L.H.S of (29)

$$2(p,(r,p,(a,b,u))] + 2([p,r],p,(b,a,u)) = (p,(u,a,b),p,r) - ((u,a,b),(p,p,r)) + 2([p,r],p,(b,a,u)]$$

$[p,(u,a,b),p,r)] - [(u,a,b),(p,p,r)] = 2(p,(r,p,(a,b,u)]$ ----------------------------from(31)

Now take each term for calculation in (31)

$[p,(u,a,b),p,r] = 2[p,(r,p,(u,a,b)]$ ---from (11)

$= 4[p,(r,p,(b,a,u)]$

$= 4[p,(p,r,(a,b,u)]$

$-[(u,a,b),(p,p,r)] = 2[(p,p,r)(b,a,u)]$ --from (13)[1]

$= -2[(p,p,r)(a,b,u)]$

$2[p,(r,p,(a,b,u)] = -2[p,(p,r,(a,b,u)]$ --from(21)[1]

Now substituting these values in (31)

$4[p,(p,r,(a,b,u)] - 2[(p,p,r)(a,b,u)] = -2[p,(p,r,(a,b,u)]$

$6[p,(p,r,(a,b,u)] = 2[(p,p,r),(a,b,u)]$

$3[p,(p,r,(a,b,u)] = [(p,p,r),(a,b,u)].$

Lemma1: If Ris a right alternative ring with $p(qr) - q(pr) \in U$ then $(R,R,U) \subseteq U$ then the ideal generated by $(R,R,U) \in R$ is $\langle (R,R,U) \rangle = (R,R,U) + (R(R,R,U).$

Proof: By the hypothesis $p(qr) - q(pr) \in U$

$$(pq)r-(p,q,r)+(q,p,r)-(qp)r\in U$$

$$-(p,q,r)+(q,p,r)+[p,q]r\in U$$

by using semi jacobian identity ,we get

$$[pr,q]+p[r,q]+[p,r,q]\in U$$

$$-[pr,u]+p[r,u]+[p,r,q]\in U$$

Using the definition of commutative centre U

$$\Rightarrow (p,r,u)\in U \;\therefore (R,R,U)\subseteq U$$

$$\Rightarrow (R,R,U)=R(R,R,U)$$

$$R(R(R,R,U))\subseteq (R,R,(R,R,U))+R^2(R,R,U)\subseteq (R,R,U)+R(R,R,U)$$

using this and (2)

$$(R,(R,R,U))R\subseteq (R,(R,R,U),R)+R((R,R,U),R)$$

$$\subseteq (R,R,(R,R,U))+R(R(R,R,U))$$

$$\subseteq (R,R,U)+R(R,R,U)$$

Lemma2; Let R be a right alternative ring with characteristic ≠2 such that $(R,R,U)\subseteq U,$ then

$k=\{p\in R/(p,R,U)=p(R,R,U)=0\}$ is an ideal of R such that $k<(R,R,U)>=0$ and

$$[[R,R],R]\subseteq K$$

proof:Let $p\in k$ using $U\subseteq N_\lambda$ --and(2)[1]

$$(q,p,r)+(q,r,p)=0$$

Since $(P,R,U)=(R,P,U)=(R,U,P)=0$ and $(R,R,U)\subseteq U$

Now we have $(pR)(R,R,U)=p(R,(R,R,U))=p((R,R,U),R)=(p(R,R,U))R=0$

And $(Rp)(R,R,U)=R(p(R,R,U))=0$

Now $(R,R,U)\subseteq U$ which means 0= $(R,[p,R]U)=([p,R],R,U)$ --------------------------using (18)

From $U\subseteq N_\lambda$ ---and using (7)

$$(pR,R,U)=(Rp,R,U)$$

$$\subseteq R(p,R,U)+(R,R,U)p+(R,p,[R,U])=p(R,R,U)=0$$

Thus it follows that Kis an ideal of R using this concept and lemma (1) we have $(R,R,U)\subseteq U$

$$K\langle(R,R,U)\rangle = K\{(R,R,U)+R(R,R,U)\} = (KR)(R,R,U)\subseteq K(R,R,U)=0$$

That leads $K\langle(R,R,U)\rangle = 0$

Final aim to show $[[R,R],R]\subseteq K$

From (18) and $U\subseteq N_\lambda$ we have

$$([[R,R],R],R,U) = -(R,[[R,R],R],U)=0$$

then by using (3) $(R,R,U)\subseteq U$ from (18) &(7),we see

$$[[R,R],R](R,R,U)\subseteq[[R,R](R,R,U),R]+[R,R][(R,R,U),R]+2([R,R],(R,R,U),R)+(R,[R,R],(R,R,U))$$

$$=[[R,R](R,R,U),R]$$

$$\subseteq[([R,R]R,R,U),R]+[([R,R],R,U)R,R]+[([R,R],R,[R,U]),R]$$

$$=[([R,R],R,U)R,R]=0$$

Lemma3:Let R be a right alternative ring with characterstic≠2,3 such that $(M,[R,R],U)=0$ then <Alt>

$<(R,[R,R],U)>0=0$ where $<(R,[R,R],U)>$ is the ideal generated by $(R,[R,R],U)$ in R

Proof: Since from(18) $(R,[R,R],U)\subseteq U\subseteq N_\lambda$

i.e $(R,[R,R],U)R\subseteq N_\lambda$,Next by using (22)

$$3(c,a,(a,[R,R],U)) = ((a,a,c),[R,R],U)=0$$

Hence $(c,a,(a,[R,R],u))=0$ since the characterstic≠3,then linearization of this identity gives

$$(R,[R,R],(R,[R,R],U)) = -(R,R,([R,R],[R,R],U))=0$$

$(R,R,(R[R,R],U))\subseteq U$ From (17)

Now by using (18)&(15)

$$(M,R,(R,[R,R],U)) = -(R,M,(R,[R,R],U)) = R,R,(M,[R,R],U)=0$$

Let $W_1=(R,[R,R],U)$ using induction we write $W_n=(R,R,W_{n-1})$ for n>1

Thus it is clear that $W_1,W_2\subseteq U$, $W,R\subseteq N_\lambda$ and

$(M,R,W_{n-2}) = ([R,R],R,W_{n-2})=0$ for some $n\geq 3$

Thus $W_{n-1,}W_n \subseteq U, W_{n-1}R \subseteq N_\lambda$

$$\therefore W_{n-1} \subseteq U \subseteq N_\lambda \subseteq ((a,a,b)R,R,W_{n-2}) + ((a,a,b),R,W_{n-2})R + ((a,a,b),R,[R,W_{n-2}]) = 0$$

$3((a,(a,R,W_{n-2}) = ((a,a,c),R,W_{n-2}) = 0$ from (22)

Since the characterstiv ≠3, $\Rightarrow (R,a,(a,R,W_{n-2})) = 0$

Then from linearization of this identity and $W_{n-1} \subseteq U \subseteq N_\lambda$

we have $(M,R,W_{n-1}) = -(R,M,W_{n-1}) = -(R,M,(R,R,W_{n-2})) = (R,R,(M,R,W_{n-2}) = 0$

similarly $([R,R],R,W_{n-1}) = -(R,[R,R],W_{n-1}) = -(R,[R,R],(R,R,W_{n-2}) = (R,R,([R,R],R,W_{n-2})) = 0$

hence $W_{n-1} \subseteq U$ from(15) $W_n = (R,R,W_{n-1}) \subseteq U$

then by induction each $W_k \subseteq U \ \& \ W_k R \subseteq N_\lambda$

thus the ideal generated in the right alternative ring R is

$(R,[R,R],U)$ is $\langle (R,[R,R],U)\rangle = \sum W_k + \sum W_k R \subseteq N_\lambda$

Thus<Alt> $< (R,[R,R],U)\rangle = 0$

MAIN RESULTS: Let R be a prime right alternative ring with characteristic ≠2,3 if

(1) $(R,R,U) \subseteq U$

(2) If $[R,R] \subseteq N_\alpha$ then U=C

(3) $(M,[R,R],U) = 0$

(4) $S(p^2,p,q) = 0$ then U=C for (2) and the remaining U=C or R is strongly (-1,1) ring.

Proof:(1) from lemma (1) we have $(R,R,U) \subseteq U$

Since $\langle (R,R,U)\rangle = 0$ and $[[R,R],R] \subseteq K$

Since R is prime, either K=0 or $\langle (R,R,U)\rangle = 0$

If k=0 then $[[R,R],R] = 0$

if $\langle (R,R,U)\rangle = 0,$ then $U \subseteq U \cap N_\gamma = C$ from def of C

then either U=C or R is strongly Ris strongly (-1,1).

(2) since $U \subseteq N_{\lambda}$ we have $(R,[R,R],U) = -([R,R],R,U) = 0$

And so $(R,R,U) \subseteq U$ from (18) then by theorem (1) either U=C or R is strongly (-1,1) but if R is a prime(-1,1) ring with characteristic≠ 2,3 and $[R,R] \subseteq N_{\alpha}$

Then R is associative by (9),since in any associative ring U=C.

(3) Since R is prime, by lemma (3)

We either have $< Alt >= 0$ or $< (R,[R,R],U) >= 0$

Now in any alternative ring with characteristic ≠3,U=C

On the other hand if $(R,[R,R],U) = 0$ then $(R,R,U) \subseteq U$ by (18)

Then by theorem(1) either again U=C or R is strongly (-1,1) ring .

4) Using $(2)^1$,(6)&(20)

$$0 = s(p^2, p, q) = (p^2, p, q) + (p, q, p^2) + (q, p^2, p)$$

$$= (p^2, p, q) - (p, p^2, q)$$

$$= -(p, p, pq) + p(p, p, q)$$

$$= [p(p, p, q)]$$

then linearization of this identity together with (12) linearized $(21)^1$ &$(2)^1$ gives

$$[(a,b,u),(p,p,u)] = -[p,((a,b,u),p,q)] - [p,(p,(a,b,u),q)]$$

$$= 3[p,(p,q,(a,b,u)]$$ Where $u \in U$

Then by (23) we have $[(a,b,u),(p,p,q)] = [(p,p,q),(a,b,u)]$

Since characteristic ≠3, gives $[(p,p,q),(a,b,u)] = 0$

But then $(M,[R,R],U) = 0$ from (15) so from the theorem (3) either U=C or R is strongly (-1, 1) ring.

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